Program of COSA2024- Conference

Monday

09:30 - 10:00 : Opening

10:00 - 10:30 : Coffee Break

10:30 - 11:30 : **Belmesnaoui Aqzzouz** (Mohamed VI University, Morocco)

Titel: General Economic Equilibrium : Model in Infinite Dimensional Vector Lattices

11:30 - 12:30 : **Anthony Wickstead** (Queen's University Belfast)

Titel: Riesz completions of some spaces of regular operators

12:30 - 14:30 : Lunch

14:30 - 15:30 : **Youssef Azouzi** (University of Carthage, Tunisia)

Titel: Aspects of compleness in Riesz spaces

15:30 - 16:00 : **Marwa Masmoudi** (University of Carthage, Tunisia)

Titel: Some characterizations of ergodicity in Riesz spaces

16:00 - 16:30 : Coffee Break

16:30 - 17:00 : **Asmae Ben rjab** (University of Tunis El Manar, Tunisia)

Titel: The strong nakano property in banach lattices

Tuesday

09:00 - 10:00 : **Christian Budde** (University of the Free State, South Africa)

Titel: Positive Miyadera-Voigt perturbations of bi-continuous semi groups

10:00 - 10:30 : **Ronalda Benjamin** (Stellenbosch University, South Africa)

Titel: The order Lozanovsky spectrum of a positive operator

10:30 - 11:00 : Coffee Break

11:00 - 12:00 : **Jamel Jabir** (University of Carthage, Tunisia)

Titel: Approximately Unital Banach lattice algebra.

12:00 - 12:30 : Sanae Boumnidel (University of Abdelmalek Essadi, Morocoo)

Titel: Some properties of weak* Dunford-Pettis operators

12:30 - 14:00 : Lunch

14:00 - 15:00 : Anki Kalauch (TU Dresden, Germany)

Titel: Generalizations of Riesz homomorphisms in order unit spaces

15:00 - 15:30 : Janko Stennder (TU Dresden, Germany)

Titel: A localization principle in pre-Riesz spaces

15:30 - 16:00 : Youssef Ezzaki (ENSA-Kenitra, Morocco)

Titel: Some results on various types of compactness of weak* Dunford-Pettis operators on Banach lattices

16:00 - 16:30 : Coffee Break

16:30 - 17:00 : Houda Moktafi (Ibno Zohr University, Morocco)

Titel: Some results on unbounded norm convergence in the bidual of a Banach lattice.

17:00 - 17:30 : Mostafa Abaali (ENSA-Kenitra, Morocco)

Titel: Modelling and analysis of the biodenitrification process

Wednesday

09:00 - 10:00 : Konstantin Dyakonov (ICREA and University of Barcelona)

Titel: Inner functions as strongly extreme points: stability phenomena

10:00 - 10:30 : Hicham Arroussi (UNIVERSITY OF HELSINKI(FI) AND READING (UK))

Titel: TOEPLITZ OPERATORS ON BERGMAN SPACES WITH EXPONENTIAL WEIGHTS

10:30 - 11:00 : Coffee Break

11:00 - 12:00 : Anil Kumar (NISER, Bhubaneswar, India)

Titel: On the geometry of an order unit space

12:00 - 12:30 : Mohamed Berka (ENSA-Kenitra, Morocco)

Titel: Some results on the Komlos sets and applications

12:30 - 14:00 : Lunch

Excursion

Tursday

09:00 - 10:00 : Emmanuel Lepinette (Paris Dauphine University)

Titel: Alternative approach to price European options in general discrete-time models without no-arbitrage condition

10:00 - 10:30 : Dorsaf Cherif (University Tunis El Manar)

Titel: Conditional indicators applied to arbitrage theory and pricing in mathematical finance

10:30 - 11:00 : Coffee Break

11:00 - 12:00 : Tahir Choulli (University of Alberta, Canada)

Titel: Reflected Backward Stochastic Differential Equations (RBSDEs) arising from pricing and hedging in models with random horizon

12:00 - 12:30 : Mohammed Mouniane (FS-Kenitra, Morocco)

Titel: On the linear moment problems and recursiveness

12:30 - 14:00 : Lunch

14:00 - 15:00 : Brus Watson (University of the Witwatersrand, South Africa)

Titel: Stochastic mixing processes in Riesz spaces

15:00 - 15:30 : Redouan Nouira (CRMEF-Kenitra, Morocco)

Titel: Some results on almost L-weakly and almost M-weakly compact operators

15:30 - 16:00 : Coffee Break

Friday

09:00 - 10:00 : Cormac Walsh

Titel: Order isomorphisms and antimorphisms in partially-ordered vector spaces

10:00 - 10:30 : Winchi Kuo (University of the Witwatersrand, South Africa)

Titel: Stochastic mixing processes in Riesz spaces

10:30 - 11:00 : Coffee Break

11:00 - 12:00 : Pedro Tradacete (Instituto de Ciencias Matemáticas Consejo Superior de Investigaciones Científicas)

Titel: Bases in Banach lattices

12:00 - 12:30 : Mourad Berraho (FS-Kenitra, Morocco)

Titel: The equi-continuity and the uniform convergence of some definable families of functions in some locally o-minimal structures

Closing of the conference

12:30 - 14:00 : Lunch

BOOK OF ABSTRACTS

Conference on ordered structures and applications

COSA 2024

ENSA, Ibn Tofail University

Kenitra, Morocco

February 05-09, 2024

General Economic Equilibrium : Model in Infinite Dimensional Vector Lattices

B. Aqzzouz
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In this talk, we attempt to give a lecture on the extension of the classical general equilibrium model on the finite dimensional linear space $\mathbb{R}^n$ to infinite dimensional vector lattices.
This problem started with Walras, then comes the contributions of Debreu-Arrow, Negishi and Debreu-Scarf in the case of finite dimensional linear spaces.
The contributions in infinite dimensional vector spaces are given by Peleg-Yaari on $\mathbb{R}^\infty$, Bewley on l^∞, Mas-Colell on vector lattice where the interior of the cone is not necessary no empty, Kreps on ordered locally convex spaces and Aliprantis - Brown on Riesz dual system (E, E').
We will emphasize the original Aliprantis - Brown contribution on the order ideal of a commodity space which is a vector lattice.

Riesz completions of some spaces of regular operators

Anthony Wickstead
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We identity the Riesz completions (in the sense of van Haandel) of some spaces of regular operators as explicitly described sublattices of the regular operators into a slightly larger range space.

Bases in Banach lattices

Pedro Tradacete
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We will survey some recent result about bases in Banach lattices. In particular, we will revisit the connections with positivity and order convergence, as well as the relation between a basic sequence and the sequence of its absolute values.

Alternative approach to price European options in general discrete-time models without no-arbitrage condition.

LÉPINETTE Emmanuel
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In mathematical finance, the problem of hedging is very classical. The well-known results that characterize the minimal super-hedging prices of a European option are established thanks to dual elements and arguments from convex analysis. They are valid for conic models that can also be defined thanks to a random preorders. But, as soon as we consider more realistic models that are not conic, duality arguments can not be applied so that a new approach is developed for discrete time models.

Inner functions as strongly extreme points : stability phenomena

Konstantin Dyakonov
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Given a Banach space, the extreme points of its unit ball are of traditional interest. Among these, the so-called strongly extreme points are also worth studying. In H^∞, the strongly extreme points are known to be the inner functions, while the (usual) extreme points are the unit-norm functions for which the appropriate logarithmic integral diverges. We show that similar characterizations remain valid for subspaces of finite codimension in H^∞. Also, we find out to what extent a non-inner function can differ from a strongly extreme point.

Generalizations of Riesz homomorphisms in order unit spaces

Anke Kalauch
Faculty of Mathematics, TU Dresden, Germany
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Riesz homomorphisms on vector lattices have been generalized to similar notions on ordered vector spaces in different ways. Riesz* homomorphisms were introduced by van Haandel using a condition on sets of finitely many elements. He attempted to prove that it suffices to take sets of two elements. We show that this is not true, in general. The description by two elements motivates to introduce mild Riesz* homomorphisms. We investigate properties of Riesz homomorphisms, Riesz* homomorphisms, mild Riesz* homomorphisms, and positive disjointness preserving operators on order unit spaces. Hereby, the geometry of the dual cone of the underlying space plays a crucial role.

The talk is based on joint work with F. Boisen, V.G. Hölker, J. Stennder, and O. van Gaans.

Approximately Unital Banach lattice algebra.

Jamel Jaber
IPEST, University of Carthage, Tunis-El Manar University, Tunisia
Email : jamel.jaber@free.fr,

Many Banach lattice algebras which occur naturally are not unital. However,Many of them do contain a positive bounded sequence or net of elements which behaves like a multiplicative identity in the limit. Such algebras are termed approximately unital Banach lattice algebras. A fundamental example of approximately unital Banach lattice algebra is found in the $C_0(X)$-type algebras or in the L^1-type ones. Addressing an open problem, we provide a positive answer to whether a Banach lattice algebra with contractive approximate identities is, indeed, an approximately unital Banach lattice algebra. Furthermore, we establish that the Fremlin projective tensor product of two Banach lattice algebras A and B, is approximately unital, if and only if A and B are approximately unital.
In addition, we investigate a representation of an approximately unital Banach f-algebra.

On the geometry of an order unit space.

Anil Kumar Karn
School of Mathematical Sciences National Institute of Science Education and Research, Bhubaneswar
Email : anilkarn@niser.ac.in,

We introduce the notion of skeleton with a head in a non-zero real vector space. We prove that skeletons with a head describe order unit spaces geometrically. Next, we prove that periphery consists of boundary elements of the positive cone of norm one. We discuss some elementary properties of the skeleton. We also find a condition under which V contains a copy of ℓ_∞^n for some $n \in \mathbb{N}$ as an order unit subspace.

Order isomorphisms and antimorphisms in partially-ordered vector spaces.

Cormac Walsh
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An order isomorphism is a bijective map that is order-preserving in both directions. In many partially-ordered vector spaces, order isomorphisms are automatically affine. Whether or not this is the case for a particular space depends on the structure of its cone of positive elements. In this talk, I will survey results in this area.
We also consider antimorphisms, where now the order is reversed. The interesting case is where the antimorphism maps the open cone of strictly positive elements to itself. Recent work suggests that the existence of such an antimorphism implies strong properties of the space. Indeed, a conjecture of Lemmens-Roelands-van Imhoff makes an intriguing connection with Jordan algebras.

TOEPLITZ OPERATORS ON BERGMAN SPACES WITH EXPONENTIAL WEIGHTS.

Hicham Arroussi (JOINT WORK WITH XIAOFEN LV)
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Given $0 < p < \infty$, the Bergman space A^p_φ consists of all holomorphic functions on $\mathbb{D}$ such that

$$\|f\|_{p,\varphi} = \left(\int_{\mathbb{D}} |f(z)e^{-\varphi(z)}|^p dA(z) \right)^{\frac{1}{p}} < \infty,$$

where φ belongs to a large class $\mathcal{W}_0$ which cover those defined by Borichev, Dhuez and Kellay (in [J. Fucnt. Anal. 242(2007), 563-606]). For $0 < p < 1$, A^p_φ is neither a self-adjoint nor Banach space. By new approaches, we study the characterizations on positive Borel measures μ on $\mathbb{D}$ for which the induced Toeplitz operators T_μ are bounded or compact from one Bergman space A^p_φ to another A^q_φ for p or $q \in (0, 1]$.

Some characterizations of ergodicity in Riesz spaces.

Marwa Masmoudi
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A great part of the theory of measure preserving systems is dedicated to the study of ergodic transformations. This property was extended to the setting of Riesz spaces by Homann, Kuo, and Watson. In this talk, we present a comprehensive study of ergodicity for a conditional expectation preserving system (E, T, S, e) and provide different characterizations of this fundamental concept in the measure-free framework of Riesz spaces. As an application, we prove that, S acts as an isometry from $L^p(T)$ to itself when $L^p(T)$ is equipped with its vector–valued norm $||.||_{p,T}$..
This talk is based on a joint work with Y. Azouzi.

Some results on unbounded norm convergence in the bidual of a Banach lattice.

Moktafi Houda
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We shall establish the stability of unbounded absolute weak convergence (uaw-convergence) under passing from sublattices. Following that, we will study the closedness and the completeness for the uaw-topology. Finally, we give an important approximation property and sequence characterizations of KB-spaces by using the uaw-convergence and the un-convergence in the bidual of a Banach lattice. As consequences, we will present some interesting characterizations of a discrete KB-space, reflexive space and discrete reflexive space by using the concept of weak convergence, un-convergence in the bidual and uaw-convergence.

The order Lozanovsky spectrum of a positive operator.

Ronalda Benjamin*, *Stellenbosch University, South Africa*
Christian Budde, *University of the Free State, South Africa* Email : ronalda@sun.ac.za ,

In studying further connections between 'Fredholm theory' and 'ordering' in the setting of bounded linear operators on Banach lattices, Alekhno introduced the Lozanovksy spectrum of a positive operator in [1]. Since the non-emptiness of this spectrum is not immediately clear, he asked and investigated in [2] whether the Lozanovsky spectrum of a positive operator defined on an arbitrary complex Banach lattice always contains the (non-empty) Weyl spectrum of the operator. To date, this remains an open question.
In this talk we will discuss the order-counterparts of the aforementioned two spectra in the context of regular operators on complex Banach lattices and show that the order Lozanovsky spectrum of a positive operator generally contains its order Weyl spectrum, recovering Alekhno's result in [2] - which gives an affirmative answer to his question for the classical Banach lattices - and extending a result from [3].

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A localization principle in pre-Riesz spaces

Janko Stennder
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In Archimedean vector lattices, it is well known that every principle ideal can be represented as a uniformly dense sublattice of some $\mathrm{C}(K)$ space. Therefore, by restricting to a suitable principle ideal, calculations can be done concretely using continuous functions. We introduce and investigate a similar localization principle in pre-Riesz spaces, involving the functional representation of order unit spaces. As an application, we prove an extension and factorization result for n-Riesz* homomorphisms on Archimedean pre-Riesz spaces.

Stochastic mixing processes in Riesz spaces

Bruce Watson
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This survey talk takes the listener through the Riesz space formulations of various martingale generalizations in the Riesz space setting. It covers martingales, Markov processes, asymptotic martingales and mixingales and their relationships.

Stochastic mixing processes in Riesz spaces

Wenchi Kuo
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Mixing processes in Riesz spaces have been a topic of recent interest. These have focused on the Riesz space analogue of measure preserving systems (conditional expectation preserving systems) - the dynamical systems formulation of mixing. Some years ago Kuo, Rogans and Watson considered the concept of statistical mixing in the Riesz space setting, generalizing the concepts of α and φ mixing. In this talk takes the audience through the essentials of statistical mixing in the Riesz space setting.

SOME PROPERTIES OF WEAK star DUNFORD-PETTIS OPERATORS.

Sanae BOUMNIDEL
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Dunford-Pettis operators are linear operators on Banach spaces that satisfy a certain property, namely that they map weakly convergent sequences to norm convergent sequences. The concept of Dunford-Pettis operators is important in the theory of Banach spaces and functional analysis.
Many interesting operators have been defined in the field of functional analysis based on this famous operator, and the weak* Dunford-Pettis is one of them. We study relationship between weak* Dunford-Pettis operators and other classes of operators on Banach lattices. After that, we establish certain properties of Banach Lattices.

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The strong nakano property in banach lattices.

Asma Ben Rejeb
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In the Banach lattice setting there is a number of relations between norm boundedness of a set and existence of suprema or upper bounds with specific properties. In addition to the classical notions of order (or Dedekind) completeness there is the Fatou property and the Nakano property.In this talk we present a stronger version of the Nakano property wich is The strong Nakano property : A Banach lattice X is said to have the strong Nakano property if for every norm-bounded upwards directed subset $A \subset X_+$ there is an upper bound $b \in X_+$ such that

$$\|b\| = \sup\{\|a\| : a \in A\}.$$

We will study at first some basic properties of Banach lattices with the strong Nakano property after that we will focus our analysis on the case of free Banach lattices.
This talk is based on a joint work with Pedro Tradecete and Y. Azouzi.

The equi-continuity and the uniform convergence of some definable families of functions in some locally o-minimal structures

Mourad Berraho
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In this talk, we first state some useful definitions to give an extension property of an equi-continuous definable function in a given DCULOAS structure, and also to study the uniform convergence for a definable family of functions in a DCULOAS expansion of an ordered field (DCULOAS is the Acronym of "definably complete uniformly locally o-minimal expansion of the second kind").

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Some results on unbounded absolute weak convergence

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In this paper, we establish the stability of uaw-convergence under passing from sublattices. The various implications of this fact are presented through the paper. In particular, we show that if (x_α) is an increasing net in a Banach lattice E and $x_\alpha \xrightarrow{uaw} 0$ in E then $x_\alpha \xrightarrow{un} 0$ in E''. Furthermore, we deduce some results concerning uaw-completeness. Additionally, we present a new characterizations of KB-spaces (resp. reflexive Banach lattices), using the concepts of uaw-convergence and un-convergence.

Positive Miyadera–Voigt perturbations of bi-continuous semigroups

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Various models of physical processes ask for positive solutions in order to have a reasonable interpretation, e.g., consider solutions containing the absolute temperature or a density. The maximum principle for elliptic and parabolic partial differential equations guarantees positive solutions under positive initial data. This demonstrates the importance of positivity in the theory of operator semigroups on Banach lattices, hence in the theory of linear evolution equations. An extensive discussion of positive operators and positive semigroups on Banach lattices can for example be found in the monographs by Arendt et al. [1], Arlotti and Banasiak [2] or Bátkai, Kramar-Fijavž and Rhandi [3].

Markov processes associated to stochastic differential equations or jointly continuous flows on metric spaces give rise to semigroups which are in general not strongly continuous with respect to the Banach space norm, cf. [Sect. 2.5, 7] and [Sect. 3.2, 6], but they do enjoy strong continuity with respect to a weaker additional locally convex topology on the Banach space. The theory of bi-continuous semigroups was introduced by Kühnemund [6] and was further studied by Farkas [5].

This work is inspired by a result due to Desch [4] and Voigt [8], where positive operator semigroups and perturbations are combined. In this talk we will discuss positive perturbations of generators of bi-continuous semigroups in the style of Voigt's work. As an application we will also consider rank-one perturbations and bi-continuous semigroups on the space $\mathrm{M}(\Omega)$ of bounded Borel measures in connection with differentiable measures.

References

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Aspects of compleness in Riesz spaces

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The talk explores various completeness concepts in vector lattices. We consider a Dedekind complete vector lattice E with a weak order unit e and a conditional expectation operator T. The space $L^p(T)$, $p \in [1, \infty]$, is furnished with a vector-valued norm, echoing the classical scenario where T assumes the role of the expectation operator. The fundamental inquiry into the completeness of these spaces with respect to this norm is far from trivial ; indeed, it constitutes the central focus of our discussion. Our primary contribution, presented in this talk, represents the fourth installment in the exploration of this subject. Initially, Kuo, Rodda, and Watson in [6] achieved a pivotal result, proving the sequential strong completeness of $L^1(T)$ and the straightforward full completeness of $L^\infty(T)$. It's noteworthy that while the latter proof is relatively straightforward, the former relied on a clever and distinct idea, diverging significantly from the classical approach. Traditional proofs for the classical case do not seamlessly transfer to establishing the sequential completeness of $L^1(T)$ in the general setting of Riesz spaces. The second work, authored by the presenter, extends the scope to sequential completeness for other values of p. Building upon an earlier result in [6] and the concept of T-uniformity, a notion generalizing uniform integrability to the setting of Riesz spaces. More recently, Kalauch, Kuo, and Watson tackled the challenge of establishing strong completeness of $L^2(T)$, employing a novel technique based on a Hahn-Jordan type Theorem. Given the absence of a direct extension of the proof from the classical case, innovative methods are crucial for success. Our current contribution aims to complete the picture, focusing on proving the completeness of other cases. This achievement stems from the strong completeness of $L^2(T)$ established in [5] and a theorem we proposed, asserting that, for any $p \in (1, \infty)$, the completeness of $L^p(T)$ is equivalent to the completeness of $L^1(T)$. A second important result that will be presented states that the space E^u is complete for the convergence in T-conditionally probability—a reasonable generalization of convergence in probability from classical theory. The theorem is then applied to establish ergodicity results for conditional preserving systems, contributing to ongoing advancements in the field.

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Reflected Backward Stochastic Differential Equations (RBSDEs) arising from pricing and hedging in models with random horizon

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We consider the model given by the quadruplet $(\Omega, \mathcal{G}, \mathbb{F}, \tau, P)$. Here $(\Omega, \mathcal{G}, P)$ is a complete probability space, $\mathbb{F}$ is a filtration defined on this space and represents the flow of information available to all agents throughout time, and τ is an arbitrary random time. This random time, which might not be observable via $\mathbb{F}$, represents default time of a firm in credit risk, the death time of an insured in life insurance where mortality and/or longevity risks pose serious challenges, the job's termination time of an *employee-stock-option*'s holder (called ESO hereafter) in finance, ..., etcetera. Hence, our current setting covers these three aforementioned frameworks and beyond. As random times can not be observable before their occurrences, the flow $\mathbb{G}$ that incorporates all the information, whether it is public or not, follows from the progressive enlargement of $\mathbb{F}$ with τ. For simplicity only, we assume that $\mathbb{F}$ is generated the Brownian motion W. In this setting, our analysis —to various hedging and pricing problems for claims built on the occurrence or not of τ— yields the following family of reflected backward stochastic differential equations (RBSDE hereafter)

$$\begin{cases} dY_t = f(t, Y_t, Z_t)d(t \wedge \tau) + g(t, Y_t, Z_t)dU_t^\tau + Z_t dW_t^\tau + dM_t - dK_t, \quad Y_\tau = \xi, \\ Y \geq S \quad \text{on} \quad 0, \tau, \quad \displaystyle\int_0^\tau (Y_{s-} - S_{s-})dK_s = 0 \quad P\text{-a.s.}. \end{cases} \tag{1}$$

Here U is a RCLL nondecreasing and $\mathbb{F}$-adapted process, while (f, g, ξ, S) is the data quadruplet where f and g are the drivers functionals, ξ is the terminal condition and S is the barrier process. It is important to mention that, for arbitrary τ, W^τ is not a martingale, and this is one of many challenges that one can encounter when addressing (1). Herein, we focus on answering the following problems :

1) What are the sufficient minimal conditions on the data (f, ξ, S, τ) that guarantee the existence of the solution of the $\mathbb{G}$-RBSDE in L^p $(p > 1)$?

2) How (1) can be **explicitly** connected to an RBSDE in $\mathbb{F}$? We want to explicitly determine the relationship between the two data triplets and between the solutions of the two RBSDEs.

3) How can we estimate –in norm– the solution $(Y^{\mathbb{G}}, Z^{\mathbb{G}}, M^{\mathbb{G}}, K^{\mathbb{G}})$ in terms of the data-triplet ? What are the adequate norms and adequate spaces for both the solution and the data-triplet ?

In my talk, I will answer all these questions deeply and beyond. Importantly, I will prove that for any random time, having a positive Azéma supermartingale, there exists a positive discount factor $\widetilde{\mathcal{E}}$ –a positive and non-increasing $\mathbb{F}$-adapted and RCLL process– that is vital in answering our questions without assuming any further assumption on τ. In particular, this will allow us to determine the adequate space for the triplet-data (f, ξ, S) and the adequate space for the solution of the RBSDE when τ is unbounded. This talk is based on the following joint works :

[1] S. Alsheyab and T. Choulli (2021) : Reflected backward stochastic differential equations under stopping with an arbitrary random time ,Preprint available on Arxiv.

[2] S. Alsheyab and T. Choulli (2023) : Optimal stopping problem under random horizon : Mathematical structures and linear RBSDEs, Preprint available on Arxiv.

Conditional indicators applied to arbitrage theory and pricing in mathematical finance.

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We introduce a large class of so-called conditional indicators, on a complete probability space with respect to a sub σ-algebra H. We study various properties of this notion which can be viewed as a generalization of the conditional expectation starting from the following property of the latter : $essinf_H X \leq E(X|H) \leq essup_H X$. The main idea is to replace $E(X|H)$ by an H-measurable mapping. Beyond the definitions, some nontrivial examples used in finance are provided. The one we focus on the most is the conditional supremum. Such a mapping is non linear has the tower property and the projection property. Several characterizations are formulated. Adding other properties of this mapping we develop an abstract theory. It inspired us for new developments in the theory of operators on Riesz spaces. After the detailed purely mathematic study of these indicators, we use them to characterize the financial market . A general framework is developed for continuous-time financial market models defined from simple strategies through conditional topologies that avoid stochastic calculus and do not necessitate semimartingale models. We then compare the usual no-arbitrage conditions of the literature, e.g. the usual no-arbitrage conditions NFL, NFLVR and the recent AIP condition. With appropriate pseudo-distance topologies, we show that they hold in continuous time if and only if they hold in discrete time. We propose the pseudo-distance : $d(X,Y) = E(esssup_H((X-Y)^+) \wedge 1), \quad X, Y \in L^0$. We study various aspects related to this distance, including the topology generated by it, convergence, etc. Furthermore we show that, in discrete-time, it is possible to evaluate the minimal super-hedging price when we restrict ourselves to integer-valued strategies. To do so, we only consider terminal claims that are continuous piecewise affine functions of the underlying asset. We formulate a dynamic programming principle that can be directly implemented on an historical data and which also provides the optimal integer-valued strategy.

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Some results on various types of compactness of weak* Dunford-Pettis operators on Banach lattices.

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We study the relationship between weak* Dunford-Pettis and weakly (resp. M-weakly, order weakly, almost M-weakly and almost L-weakly) operators on Banach lattices. The following is one of the major results dealing with this matter : If E and F are Banach lattices such that F is Dedekind σ-complete, then each positive weak* Dunford–Pettis operator $T : E \to F$ is weakly compact if and only if one of the following assertions is valid : (a) the norms of E' and F are order continuous ; (b) E is reflexive ; (c) F is reflexive.

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Some results on almost L-weakly and almost M-weakly compact operators

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The class of weakly compact operators does not contains that of almost L-weakly compact operators. In this paper, we provide a complete answer by giving necessary and sufficient conditions for which every positive almost L-weakly compact operator $T : E \to F$ between two Banach lattices is weakly compact. On the other hand, we investigate conditions under which the adjoint operator of every positive almost L-weakly compact operator is almost M-weakly compact.

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Modelling and analysis of the biodenitrification process

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Massive and excessive use of nitrogen fertilizers and sustained irrigation have been widely practised in recent years. This strategy leads to a large transfer of nitrates to groundwater, leading to a major environmental problem of nitrate contamination in water, intended for drinking water consumption. One of the most effective solutions is the degradation of nitrites and nitrates, into gaseous nitrogen, using the heterotrophic bacteria during the denitrification process. In this paper, we present and study a mathematical model of the biodenitrification process taking into account the fixed and mobile bacteria. This process is modeled by a system of ordinary differential equations and requires the success of bacteria to colonize the reactor. We study the existence and the asymptotic behaviour of the solution. We show the existence of a value of the injected carbon concentration from which we ensure the success of the biodenitrification process and we propose a heuristic algorithm which serves to control the biodenitrification process over time. Finally, we present some numerical simulations in to support the theoretical results.

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ON THE DISJOINT WEAK BANACH-SAKS OPERATORS

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We introduce and study a new class of operators that we call disjoint weak Banach-Saks operators. We establish some characterizations of this class of operators by different types of convergence (norm convergence, unbounded order convergence, unbounded norm convergence and unbounded absolute weak convergence) as well as by the positive weakly null sequences. Consequently, we give a new characterization of the disjoint weak Banach-Saks property by the positive disjoint weakly null sequences. Furthermore, we study the relationship between this class and other classes of operators.

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On the linear moment problems and recursiveness

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In view of its fundamental role in various fields of mathematics and applied science, the linear moment problem has been extensively studied in the literature. Especially, it has been shown that this problem is useful for some topics in physics, such that the quantum dynamical systems. Recently, the linear moment problem has been investigated in the literature, by various methods. The present talk aimed to explore the linear moment problem for the real sequences defined by the nonhomogeneous linear recursive relation. Various properties are provided, especially, those related to the Hankel matrices. Some considerations in connection with K-moment problem, for the nonhomogeneous recursive, are discussed.

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