Conference Programme

Conference on Ordered Structures with Applications

COSA
Tozeur 2023

Monday		
9:00–9:30	Opening	
9:30–10:30	Anatoly Kusraev	Boolean-valued analysis and positivity
10:30–11:00	Coffee break	
11:00–12:00	Bruce Watson	Strong completeness and a Riesz representation theorem in Riesz spaces
12:00–13:30	Lunch	
13:30–14:00	Jonathan Homann	Characterisation of conditional weak mixing via ergodicity of the tensor product in Riesz spaces
14:00-14:30	Youssef Naasri	Generating functions in Riesz spaces
14:30–15:00	Marten Wortel	Representing Artinian vector lattices
15:00–15:30	Coffee break	
15:30–16:30	Valentin Ferenczi	Isometries on interpolation scales with or without lattice structure
Tuesday		
9:00–10:00	Jawad H'michane	Contribution to L-weakly compact sets
10:00-10:30	Marwa Masmoudi	Recurrence in Ergodic theory
10:30–11:00	Coffee break	
11:00–12:00	Wen Chi Kuo	Laws of large and small numbers in Riesz spaces
12:00–13:30	Lunch	
13:30–14:30	Jamel Jaber	The Fremlin projective tensor product of Banach lattice algebras
14:30–15:00	David de Hevia Rodriguez	The Complemented Subspace problem in Banach lattices
15:00–15:30	Enrique Garcìa-Sánchez	Duality in free Banach lattices
15:30–16:00	Coffee break	
16:00–16:30	Amal Omrani	Concentration inequalities in Riesz spaces
16:30–17:00	Damla Yaman	Tensor products of Riesz subspaces

Wednesday		
9:00–10:00	Emmanuel Lepinette	Dynamic programming principle and computable prices in financial market models with transaction costs
10:00–10:30	Dorsaf Cherif	No-arbitrage conditions and pricing from discrete-time to continuous-time strategies
10:30–11:00	Coffee break	
11:00–12:00	Yury Korolev	Connections between neural networks and vector lattices
12:00–12:30	Samir Smiti	Lifting component in clean vector lattices and approximation by step functions
12:30–13:30	Lunch	
13:30–17:00	Excursion	

Thursday		
9:00–10:00	Anatoly Kusraev	Stone–Weierstrass theorem
10:00–11:00	Youssef Azouzi	Inequalities in Riesz spaces
11:00–11:30	Coffee break	
11:30–12:30	Pedro Tradacete	Free Banach lattices
12:30–14:00	Lunch	
14:00–15:00	Anke Kalauch	An order theoretical analysis of atomic JBW algebras
15:00–16:00	Onno van Gaans	The order center and the algebraic center of JB-algebras
16:00–16:30	Coffee break	
16:30–17:00	Janko Stennder	Algebraic structures in pre-Riesz spaces

Friday		
9:00–10:00	Belmesnaoui Aqzzouz	Some results on portfolio theory in infinite-dimensional vector lattices
10:00–11:00	Anil Kumar Karn	Absolute compatibility in von Neumann algebras
11:00–11:30	Coffee break	
11:30–12:30	Eugene Bilokopytov	Various characterizations of projection bands
12:30–14:00	Lunch	
14:00	End of conference	

Book of Abstracts

Conference on Ordered Structures with Applications

Tunis
2023

Some Results on Portfolio Theory in Infinite-dimensional Vector lattices

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Abstract. We assume that we are in a security market with infinitely many securities. We define the portfolio space on which we will define an order and what is the supremum of two portfolios and the infimum of two portfolios for our order. The ordered vector space of portfolios that we obtain is not necessary a vector lattice in general. We will look for sufficient condition who make our space a vector lattice. Also, We will define a locally convex vector topology on our space of portfolios and define the vector space of prices, which are continuous linear forms on the space of portfolios. As we work in security market with infinite securities, the equivalent of Arrow–Debreu equilibrium does not exist in general. We look for sufficient condition for which this equilibrium exists. We will do it step by step and finally use the second welfare theorem of Arrow-Debreu to prove this existence.

Inequalities in Riesz spaces

Youssef Azouzi
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Abstract. We present a talk about inequalities in Riesz spaces. Most of them come from probability theory and have been generalized to the setiing of measure-free setting of vector lattices. Several tools are used to obtain them and several ideas are combined. Some of them lead to introduce new concepts and some of the have nice applications.

Various characterizations of projection bands

Eugene Bilokopytov
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Abstract. In this talk we will consider several different criteria for an ideal J of an Archimedean vector lattice E to be a projection band. In particular, we will show that an ideal is a projection band iff every collection of mutually disjoint elements which is order bounded in E, is also order bounded in J. Another notable criterion is given in terms of the unbounded norm topology on a Banach lattice E: a closed ideal J is a projection band iff the unbounded norm convergence of E coincides on J with the unbounded norm convergence of J.

No-arbitrage conditions and pricing from discrete-time to continuous-time strategies

Dorsaf Cherif
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Abstract. In this paper, a general framework is developed for continuous-time financial market models defined from simple strategies through conditional topologies that avoid stochastic calculus and do not necessitate semimartingale models. We then compare the usual no-arbitrage conditions of the literature, e.g. the usual no-arbitrage conditions NFL, NFLVR and NUPBR and the recent AIP condition. With appropriate pseudo-distance topologies, we show that they hold in continuous time if and only if they hold in discrete time. Moreover, the super-hedging prices in continuous time coincide with the discrete-time super-hedging prices, even without any no-arbitrage condition.

Isometries on interpolation scales with or without lattice structure

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Abstract. We shall present recent results and open questions regarding isometric actions of groups on interpolation scale. In a first part we shall present results on interpolation scales of function spaces with lattice structure, focusing on their relations with exact sequences in Banach spaces (results of Kalton and coauthors). In a second part we shall discuss what happens when the group of units of a function space is replaced by an arbitrary isometry group.

The order center and the algebraic center of JB-algebras

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Abstract. For a directed partially ordered vector space X, one can consider the vector space of all linear operators on X ordered by the cone of positive operators. The subspace of all operators that are below and above a multiple of the identity operator is called the order center of X. By the algebraic center of an associative algebra, one usually means the set of those elements that commute with all other elements. We will consider JB-algebras, which are both Jordan algebras and Banach spaces with suitably compatible norms. A JB-algebra is commutative but not associative. Endowed with the cone of squares, it is a directed partially ordered vector space. The algebraic centre of a JB-algebra X is defined to be the set of those elements whose corresponding left multiplication operator commutes with all other left multiplication operators. We will show that the order center and the algebraic center of a unital JB-algebra are isomorphic.

Contribution to L-weakly compact sets

Jawad H'michane
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Abstract. Based on the concept of unbounded absolutely weakly convergence, we give new characterizations of L-weakly compact sets. As applications, we find some properties of order weakly compact operators. Also, a new characterizations of order continuous Banach lattices are obtained. On the other hand, we study the relationship between limited (resp. (L)) sets and L-weakly compact sets, and in the last part, we give some results about the relationship between (L) sets and L-weakly compact sets.

Characterisation of conditional weak mixing via ergodicity of the tensor product in Riesz spaces

Jonathan M. Homann
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Abstract. We link conditional weak mixing and ergodicity via the tensor product in Riesz spaces. In particular, we characterise conditional weak mixing of a conditional expectation preserving system by the ergodicity of its tensor product with itself or other ergodic systems. Along the way, we characterise components of weak order units in the tensor product space in terms of tensor products of components of weak order units. This is joint work with Mohamed Amine Ben Amor, Wen-Chi Kuo and Bruce A. Watson.

The Fremlin projective tensor product of Banach lattice algebras

Jamel Jaber
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Abstract. In this work we prove that the Fremlin projective tensor product of Banach lattice algebras can be endowed with a Banach lattice algebra structure and satisfies an appropriate universal property. In addition, we prove that the Fremlin projective tensor product of Banach f-algebras is a Banach f-algebra.

An order theoretical analysis of atomic JBW algebras

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Abstract. Atomic JBW-algebras are known to be direct sums of JBW-algebra factors of type I. Extending Kadison's anti-lattice theorem, we establish that each of these factors is a disjointness free anti-lattice. We characterize order theoretical notions as disjointness, bands, and disjointness preserving bijections with disjointness preserving inverses in direct sums of disjointness free anti-lattices and, hence, in atomic JBW-algebras.

Absolute compatibility in von Neumann algebras

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Abstract. Let A be a unital C^*-algebra and let $a, b \in A^+$ with $||a|| \leq 1$ and $||b|| \leq 1$. We say that a is absolutely compatible with b if $|a - b| + |1 - a - b| = 1$.This notion generalizes algebraic orthogonality among positive elements and plays an important role in proving spectral decomposition theorem using order theoretic techniques. In this talk, we shall discuss its elementary properties and present a complete description within a von Neumann algebra. We also show that in $\mathbf{M_2}$, this property exhibits some geometric patterns which leaves hints of some possible physical interpretations.

Connections between Neural Networks and Vector Lattices

Yury Korolev
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Abstract. In this talk I will present a brief overview of the theory of neural networks with an emphasis on their approximation properties. I will discuss universal approximation by neural networks as well as quantitative estimates of the approximation error that are independent of the input dimension. I will introduce different neural network architectures, such as feedforward networks with one or multiple hidden layers and residual networks. I will also discuss generalisations to the case of Banach-space-valued neural networks and propose to think about how the theory of vector lattices might come into play.

Laws of large and small numbers in Riesz spaces

Wen-chi Kuo
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Abstract. Martingales, Markov processes and Laws of Large Numbers have been well studied in the Riesz space (vector lattice) setting. There has previously been some attention given to Laws of Large Numbers in Riesz spaces and very little to Laws of Small Numbers in Riesz spaces. A survey will be given of LLN and LSN in Riesz spaces along with the background required for these developments. Aspects of this work have been jointly developed with: J. Homann, C. Labuschagne, N. Musara, D. Rodda, J. Vardy and B.A. Watson.

Dynamic programming principle and computable prices in financial market models with transaction costs

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Abstract. How to compute (super) hedging costs in rather general financial market models with transaction costs in discrete-time ? Despite the huge literature on this topic, most of results are characterizations of the super-hedging prices while it remains difficult to deduce numerical procedure to estimate them. We establish here a dynamic programming principle and we prove that it is possible to implement it under some conditions on the conditional supports of the price and volume processes for a large class of market models including convex costs such as order books but also non convex costs, e.g. fixed cost models.

Recurrence in Ergodic theory

Marwa Masmoudi
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Abstract. In this talk, some classical results in ergodic theory dealing with recurrence are treated especially the Kac formula and Poincaré Recurrence theorem. We develop Riesz spaces generalizations of these theorems. We also present different versions of the classical Kakutani-Rokhlin lemma and study the connection between them. This talk is based on a recent joint work with Y. Azouzi, M.A. Ben Amor, J. Homann And B. Watson.

Generating functions in Riesz spaces

Youssef Naasri
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Abstract. We introduce and study the concept of generating function for natural elements in a Dedekind complete Riesz space equipped with a conditional expectation operator. This allows to study discrete processes in free-measure setting. In particular we improve a result obtained by Kuo, Vardy and Watson concerning Poisson approximation.

Concentration inequalities in Riesz spaces

Amal Omrani
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Abstract. It has been established that order is closely related to probability theory, Watson and Kuo gener- alized the conditional expectation to the frame work of Riesz spaces as positive order-continuous

projections operators mapping weak order units to weak order units and having Dedekind com- plete range. We will provide a construct of the exponential function and review some of its relevant

characteristics to the Riesz spaces. Then we will depict how this function beholds a key role in generalizing the renowned Moment Generating function and prove its most relevant properties which will be used it to prove some of the concentration inequalities in Riesz spaces.

The Complemented Subspace problem in Banach lattices

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Abstract. The *Complemented Subspace Problem in Banach lattices* asks whether any complemented subspace of a Banach lattice must be isomorphic to a Banach lattice. This question has several relevant ramifications, for instance:

- Is there any complemented subspace of $L_1[0,1]$ which is not isomorphic to ℓ_1 or $L_1[0,1]$?
- Is there any complemented subspace of $C[0,1]$ which is not isomorphic to a $C(K)$-space?

As we shall indicate in the talk, a positive answer to the Complemented Subspace Problem in Banach lattices in the separable case would also imply a positive solution to the two previous questions.

In 2021, G. Plebanek and A. Salguero-Alarcón provided an example, denoted by PS_2, of a one-complemented subspace of a $C(K)$-space which is not isomorphic to a $C(L)$-space (with L being an arbitrary compact Hausdorff space). This answered in the negative the Complemented Subspace Problem in the $C(K)$ setting. We shall show that, in fact, the space PS_2 is not even isomorphic to a Banach lattice. Nevertheless, this counterexample is non-separable so the Complemented Subspace Problem is still open in the separable case.

This talk is based on a joint work with Gonzalo Martínez-Cervantes, Alberto Salguero-Alarcón and Pedro Tradacete.

Duality in free Banach lattices

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Abstract. The free Banach lattice generated by a Banach space E is a Banach lattice $FBL[E]$, together with a linear isometric embedding $\phi_E : E \to FBL[E]$, satisfying the following universal property: for every Banach lattice X and every bounded and linear operator $T : E \to X$ there exists a unique lattice homomorphism $\hat{T} : FBL[E] \to X$ such that $\hat{T} \circ \phi_E = T$, with $\|\hat{T}\|_0 = \|T\|$. Similarly, we can define the free p-convex Banach lattice, denoted $FBL^{(p)}[E]$, by imposing that the Banach lattices X in the previous definition is p-convex. At the same time, the bidual E^{**} of a Banach space E can be understood as a free object over E in the category of dual Banach spaces with adjoint operators. In this talk we will study the interaction between free Banach lattices and free duals in two ways. On the one hand, we will see that $FBL^{(p)}[E^{**}]$, the free p-convex Banach lattice generated by the free dual over E, embeds isometrically into $FBL^{(p)}[E]^{**}$, the free dual over the free p-convex Banach lattice generated by E. On the other hand, we will combine the previous notions to show the existence of free objects in the category of p-convex dual Banach lattices. This is a joint work with Pedro Tradacete.

Lifting component in clean vector lattices and approximation by step functions

Samir Smiti
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Abstract. Let G be an abelian l-group with a strong order unit $u > 0$. We call Gu-clean after Hager, Kimber, and McGovern if every element of G can be written as a sum of a strong order unit of G and a u-component of G. We prove that G is u-clean if and only if u-components of G can be lifted modulo any l-ideal of G. Moreover, we introduce a notion of u-suitable l-groups (as a natural analogue of the corresponding notion in Ring Theory) and we prove that the l-group G is u-clean when and only when it is u-suitable. Also, we show that if E is a vector lattice, then E is u-clean if and only if the space of all u-step functions of E is u-uniformly dense in E. As applications, we will generalize a result by Banaschewski on maximal l-ideals of an Archimedean bounded f-algebras to the non-Archimedean case. We also extend a result by Miers on polynomially ideal $C(X)$-type algebras to the more general setting of bounded f-algebras.

Algebraic structures in pre-Riesz spaces

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Abstract. We consider pre-Riesz spaces together with associative multiplications given by different kinds of bilinear maps. The aim is to generalize results from the theory of lattice-ordered algebras, in particular, f-algebras, almost f-algebras, and d-algebras. Based on this, we study properties of generalizations of bi-orthomorphisms, bi-Riesz homomorphisms, and orthosymmetric maps on pre-Riesz spaces. Especially on order dense subspaces of C(K)-spaces, we established some first results. Among other things, statements regarding algebra homomorphisms, nilpotent elements, and representations are presented.

Free Banach lattices

Pedro Tradacete
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Abstract. Free Banach lattices were first introduced by B. de Pagter and A. Wickstead in 2015. Given any Banach space, a free Banach lattice can be associated to it in a canonical way. This construction produces a new link between Banach spaces and Banach lattices translating problems from one setting to the other, and raising many open questions. We will survey on recent progress about this notion.

Strong completeness and a Riesz representation theorem in Riesz spaces

Bruce A. Watson
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Abstract. A Hahn-Jordan decomposition in Riesz spaces which generalizes that of B. A. Watson (2009) will be discussed and used to obtain a Riesz-Frechet representation theorem for the T-strong dual, where T is a Riesz space conditional expectation operator. The strong sequential completeness of the space $L^1(T)$, the natural domain of the conditional expectation operator T, and the strong completeness of $L^\infty(T)$ was established in W.-C. Kuo, D. Rodda, B. A. Watson (2019). Here we discuss the T-strong completeness of $L^2(T)$. This is joint work with A. Kalauch and W. Kuo.

Representing Artinian vector lattices

Marten Wortel
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Abstract. A vector lattice is Artinian if every decreasing sequence of ideals stabilizes. We will discuss a representation theorem for Artinian vector lattices. The main tools used are lexicographic vector lattices, local and semi-local ideals, and well-founded induction.

This is joint work with Marko Kandic and Mark Roelands.

Tensor products of Riesz subspaces

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Abstract. Fremlin introcued the Riesz tensor product of vector lattices and it is known that the Riesz tensor product of two vector lattices is a vector lattice itself. It has been an open question whether the tensor product of ideals or bands of a vector lattice is also an ideal or band. It is not true for ideals as Buskes and Thorn presented a counterexample to that. Recently Grobler introduced the Dedekind complete tensor product, as the Dedekind completion of the Riesz tensor product, that is, $E\overline{\otimes}_\delta F = (E\overline{\otimes}F)^\delta$. We consider ideals and bands of a vector lattice and their Dedekind complete tensor product. It turns out that the Dedekind completion of the Riesz tensor product of two ideals is again an ideal. We also give some results for the Dedekind complete tensor product of projection bands.